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Prof. Arnaldo Gutiérrez (6024)
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Dear Professor Gutiérrez:

Thank you for your interest in the capacity spectrum method. Enclosed is a copy of the paper *The Capacity Spectrum Method for Determining the Demand Displacement* that you requested. For your information, I have just completed an updated paper on The Capacity Spectrum Method that will be published and presented at the 6th U.S. Conference on Earthquake Engineering in June 1998 in Seattle, Washington.

Very truly yours,

WISS, JANNEY, ELSTNER ASSOCIATES, INC.

[Signed]

Sigmund A. Freeman, S.E.
Principal

SAF/dlt
Enclosure
THE CAPACITY SPECTRUM METHOD
FOR DETERMINING
THE DEMAND DISPLACEMENT
by
Sigmund A. Freeman
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2200 Powell Street, Suite 925
Emeryville, CA  94608
for presentation at the

ACI 1994 SPRING CONVENTION
MARCH 23, 1994
9:00 AM - NOON
Technical Session: Displacement Considerations
in Design of Earthquake-Resisting Buildings

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The Capacity Spectrum Method was originally developed as a rapid evaluation method
for a pilot seismic risk project of the Puget Sound Naval Shipyard for the U.S. Navy (Freeman,
et al. 1975). It was later used as a procedure to correlate earthquake ground motion with
observed building performance (Freeman 1978 and ATC 1982) and has been incorporated in the
TriServices Seismic Design Guidelines for Essential Buildings (Army 1986) as part of the two-
level approach to seismic design. The Capacity Spectrum Method is not presented as a
theoretically accurate procedure, but it does have the feature of rationally explaining how
buildings respond to earthquake ground motion.

The Capacity Spectrum Method takes a graphical representation of the global force-
displacement capacity curve of the structure and compares it to the response spectrum
representation of the earthquake demands.

The capacity curve is determined by statically loading the structure with lateral forces to
calculate the roof displacement and base shear coefficient that defines first significant yielding
of structural elements. The yielding elements are then relaxed to form plastic hinges and
incremental lateral loading is applied until a nonlinear static capacity curve is created. The
curve is created by superposition of each increment of displacement and includes tracking
displacements at each story (ATC 1982). This procedure is sometimes referred to as the
pushover analysis. It is assumed that the structure can take a number of cycles along the
capacity curve and behave in a hysteretic manner. The stiffness is assumed to reduce to an
equivalent global secant modulus measured to the maximum excursion along the capacity curve
for each cycle of motion. The roof displacement and base shear coefficient coordinates are
converted to spectral displacements ($S_d$) and spectral accelerations ($S_a$) (Fig. 1), respectively, by
use of modal participation factors and effective modal weights as determined from dynamic
characteristics of the fundamental mode of the structure. These values change as the displaced
shape changes. An equivalent inelastic period of vibration ($T_i$) at various points along the
capacity curve are calculated by use of the secant modulus (i.e., $T_i = 2\pi \sqrt{S_{ae}/S_{ag}}$). Now the capacity spectrum curve can be plotted with the same coordinates as a response spectrum (i.e., $S_a$ vs $T$) (Fig. 2).

The demand curve is represented by earthquake response spectra. It is presented at various levels of damping. For example, the 5 percent damped curve may be used to represent the demand when the structure is responding elastically. The 10 percent and 20 percent damped curves may be used to represent the reduced demand in the inelastic range to account for hysteretic damping and anti-resonant nonlinear effects. The higher damped curves are used as a tool for reducing the spectrum to an equivalent inelastic spectrum and are not presented as necessarily being theoretically correct.

In order to more visually illustrate the relationship between elastic displacement demands and inelastic displacement demands, the $S_a$ vs $T$ coordinate system for the response spectra (e.g., Fig. 3) can be converted to a set of coordinates defined by $S_a$ and $S_i$ (i.e., $S_i = S_{ag} (T/2\pi)^3$) (e.g., Fig. 4). When the spectral values are plotted in this acceleration-displacement response spectrum format (ADRS), the period can be represented by lines radiating from the origin (Mahaney, et al. 1993).

For either set of coordinates (i.e., $S_a$ vs $T$ or $S_i$ vs $S_a$), both the capacity and demand curves are plotted together. The Capacity Spectrum Method can be summarized as follows:

If the capacity curve can extend through the envelope of the demand curve, the building survives the earthquake. The intersection of the capacity and appropriately damped demand curve represents the inelastic displacement of the structure.

To illustrate the Capacity Spectrum Method in the ADRS format several response spectra are presented. They include the UBC/SEAOC design earthquake spectra for $Z=0.4$ and the S2 soil profile (ICBO 91, SEAOC 90) and the Loma Prieta Earthquake recorded in downtown Oakland, California (L-P, Oak) and in downtown San Francisco (L-P, SP). These spectra are plotted for 5%, 10%, and 20% damping.

An idealized capacity curve is used that represents a code-designed reinforced concrete frame building. The initial (un-cracked section properties) period is 0.62 seconds. If the structure was strong enough to remain elastic at its initial period, $S_a = 0.98g$ and $S_i = 3.6$ inches at Point A for the 5% damped spectra (Note: at "cracked" section properties the period would be about 0.9 sec with $S_a = 0.67g$ and $S_d = 5.3$ inches). However, as seen in Figure 4, the structure starts cracking at $S_i = 0.10g$ and is exhibiting significant yielding at $S_d = 2$ inches. An ultimate capacity is estimated at Point B, where $S_a = 0.35g$ and $S_d = 7.2$ inches. The effective period, based on the global secant stiffness is $T = 1.45$ seconds. For this period of vibration, the demand of the 20% damped response spectra is $S_a = 0.25g$ and $S_d = 5.3$ inches (Pt. C). The 20% damped spectra is assumed to represent the inelastic spectra for the subject structure cycling near its ultimate capacity. Because the capacity of the structure exceeds the 20% damped spectra demand (i.e., it passes through the 20% damped spectra), it is assumed that the effective damping is somewhere between 10% and 20%. An estimate is made by constructing a line between points A and C and noting the intersection at Point D. Point D is the maximum excursion of the structure for the demand earthquake. Thus, $S_a = 0.32g$, $S_d = 5.1$ inches, $T = 1.25$ sec, and effective damping is about 16% of critical damping.
Once the capacity curve is developed, it can be used for any response spectra. For the 1989 Loma Prieta earthquake in Oakland (Fig. 5), $S_p$ is estimated at 0.3g and $S_d$ at 4 inches (i.e., somewhere between 10% and 20% damping). For San Francisco (Fig. 6), $S_p$ is estimated at 0.25g and $S_d$ at 2 inches for 10% damping. The Figure 5 spectra were developed from a ground motion recording at a 2-story building in Oakland and Figure 6 from an 18-story building in San Francisco.

This paper is an updated segment of a presentation made at the 5th U.S.-Japan Workshop on Improvement of Building Structural Design and Construction Practices, ATC 15-4, and is a follow-up on an earlier paper on correlation of code forces to earthquake demands (Freeman 92).

REFERENCES


ATC, 1982, An Investigation of the Correlation Between Earthquake Ground Motion and Building Performance (ATC-10), Applied Technology Council, Redwood City, California.


Freeman, S.A., 1978, Prediction of Response of Concrete Buildings to Severe Earthquake Motion, Douglas McHenry International Symposium on Concrete and Concrete Structures, SP-55, American Concrete Institute, Detroit, Michigan, pp 589-605.


SEAOC, 1990, Recommended Lateral Force Requirements and Commentary, Seismology Committee, Structural Engineers Association of California (Fifth Edition Revised), San Francisco, California.
Fig. 1--Spectral acceleration versus spectral displacement

Fig. 2--Spectral acceleration versus effective period

Fig. 3--Capacity and demand, spectral acceleration versus period

from Freeman, 1978
**Fig. 4.** Capacity Spectrum Method - ADRS.

**OAKLAND 2-STORY BLDG; LOMA PRIETA**

**Fig. 5.** C.S.M. - Oakland
Fig. 6. C.S.M. - San Francisco